WEEKLY TEST TARGET - JEE - 01 TEST - 12
SOLUTION Date 28-07-2019

[PHYSICS]

1. $H = \frac{v^2 \sin^2 \theta}{2g}$ and $R = \frac{v^2 \sin^2 2\theta}{g}$

Since, R = 2H, so $\frac{v^2 \sin 2\theta}{g} = 2 \times \frac{v^2 \sin^2 \theta}{2g}$

or 2sinθ cosθ = sin²θ or or tanθ = 2

$\therefore$ $R = v^2 \times \frac{2}{g} \times \sin\theta \cos\theta$

$= \frac{2v^2}{g} \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4v^2}{5g}$

2. For the preson to be able to catch the ball, the horizontal component of velocity of the ball should be same as the speed of the person, i.e.,

$v_0 \cos\theta = \frac{v_0}{2}$

or $\cos\theta = \frac{1}{2}$ or $\theta = 60^o$

3. Let, $u_x = 3$ m/s, $a_x = 0$
$u_y = 0$, $a_y = 1$ m/sec^2 and t = 4 sec
If v_x and v_y be the velocities after 4 sec respectively, then
$v_x = u_x + a_x t = 3$ ms^{-1}
and $v_y = u_y + a_y t = 0 + 1 \times 4 = 4$ ms^{-1}

$\therefore$ $v = \sqrt{v_x^2 + v_y^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = 5\ m/s$

Angle made by the result velocity w.r.t. direction of initial velocities, i.e., x-axis, is

$\beta = \tan^{-1} \frac{v_y}{v_x} = \tan^1 \left(\frac{4}{3}\right)$

4. $h = \frac{u^2 \sin^2 \theta}{2g}$, hence, $\frac{\Delta h}{h} = 2.\frac{\Delta u}{u}$

Since, $\frac{\Delta u}{u} = 2\%$, hence, $\frac{\Delta h}{h} = 4\%$

5. $R_{max.} = R = \frac{u^2}{g}$ or $u^2 = Rg$

Now, as $\text{range} = \frac{u^2 \sin 2\theta}{g}$

then, $\frac{R}{2} = \frac{Rg \sin 2\theta}{g}$

or $\sin 2\theta = \frac{1}{2} = \sin 30^o$

or $\theta = 15^o$

6. The horizontal range is the same for the angles of projection θ and $(90^o - \theta)$

$$t_1 = \frac{2u\sin\theta}{g}$$

$$t_2 = \frac{2u\sin(90^o - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\therefore \quad t_1 t_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g} = \frac{2}{g}\left[\frac{u^2 \sin 2\theta}{g}\right] = \frac{2}{g}R$$

where $R = \frac{u^2 \sin 2\theta}{g}$

Hence, $t_1 t_1 \propto R$.

7. Range = 150 = ut and

$$h = \frac{15}{100} = \frac{1}{2} \times gt^2$$

or $t^2 = \frac{2 \times 15}{100 \times g} = \frac{30}{1000}$

or $t = \frac{\sqrt{3}}{10}$

$$\therefore \quad u = \frac{150}{t} = \frac{150 \times 10}{\sqrt{3}} = 500\sqrt{3}\ \text{ms}^{-1}$$

8. $y = 12x - \frac{3}{4}x^2$

$$\frac{dy}{dt} = 12\frac{dx}{dt} - \frac{3}{2}x\frac{dx}{dt}$$

At x = 0: $\frac{dy}{dt} = 12\frac{dx}{dt}$

If θ be the angle of projection, then

$$\frac{dy/dt}{dx/dt} = 12 = \tan\theta$$

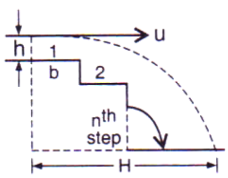
Also, if u = initial velocity, then $u\cos\theta = 3$

Hence, $\tan\theta \times u\cos\theta = 36$ or $u\sin\theta = 36$

Range, $R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin\theta\cos\theta}{g}$

$$= \frac{2(u\sin\theta)(u\cos\theta)}{10} = \frac{2 \times 36 \times 3}{10} = 21.6 \text{ m}$$

9. If the ball hits the nth step, the horizontal distance traversed = nb.
Vertical distance traversed = nh.
Here, velocity along horizontal direction = u.
Velocity along vertical direction = 0.



∴ nb = ut .....(i)

$nh = 0 + \frac{1}{2}gt^2$ ....(ii)

From eqn. (i),

$t = \frac{nb}{u}$, ∴ $nh = \frac{1}{2}g \times \left(\frac{nb}{u}\right)^2$

∴ $n = \frac{2hu^2}{gb^2}$

10. Potential energies at the highest point are equal to the loss in kinetic energies. That is $\frac{1}{2}mu^2$ and $\frac{1}{2}m(u\cos 60^o)^2$ or $\frac{1}{4} \times \frac{1}{2}mu^2$.

11. $t = \frac{2u\sin\theta}{g} = \frac{2 \times 20 \times \sin 30^o}{10} = 2\ s$

Now, we shall calculate the total time taken by the ball to hit the ground.

Using, $s = ut + \frac{1}{2}gt^2$,

we get; $40 = -10t' + \frac{1}{2} \times 10(t')^2$

[∵ u = – 20 sin30º = – 10 m/s]

∴ 5(t′) – 10t′ – 40 = 0

Solving, we have, t′ = 4 s

∴ $\frac{t'}{t} = \frac{4s}{2s} = \frac{2}{1}$

12. $H_{max.} = \frac{u^2 \sin^2\theta}{2g}$

$T = \frac{2u\sin\theta}{g}$

$\frac{H_{max.}}{T^2} = \frac{u^2\sin^2\theta}{2g} \times \frac{g^2}{4u^2\sin^2\theta} = \frac{g}{8} = \frac{10}{8} = \frac{5}{4}$

13.

14. $H = \frac{u^2\sin^2\theta}{2g}$ or $80 = \frac{u^2\sin^2\theta}{2 \times 10}$

or $u^2 \sin\theta = 1600$

or $u\sin\theta = 40\ ms^{-1}$.

Horizontal velocity = u cosθ = at

= 3 × 30 = 90 ms^{-1}

$\frac{u\sin\theta}{u\cos\theta} = \frac{40}{90}$

or $\tan\theta = \frac{4}{9}$ or $\theta = \tan^{-1}\left(\frac{4}{9}\right)$

15. $h = (u\sin\theta)t - \frac{1}{2}gt^2$

d = (u cosθ)t

or $t = \frac{d}{u\cos\theta}$

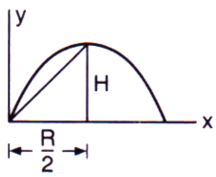
$\therefore \quad h = u\sin\theta.\frac{d}{u\cos\theta} - \frac{1}{2}g.\frac{d^2}{u^2\cos^2\theta}$

$\therefore \quad u = \frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\tan\theta - h)}}$

16. Average velocity $= \frac{\text{Displacement}}{\text{Time}}$

$v_{av.} = \frac{\sqrt{H^2 + \frac{R^2}{4}}}{T/2}$ ......(i)

Here, H = maximum height



$= \frac{v^2\sin^2\theta}{2g}$

$R = \text{range} = \frac{v^2\sin 2\theta}{g}$

and $T = \text{time of flight} = \frac{2v\sin\theta}{g}$

$\therefore \quad v_{av.} = \frac{v}{2}\sqrt{1 + 3\cos^2\theta}$

17. $T = \frac{2u_y}{g}, \qquad H = \frac{u_y^2}{2g}$

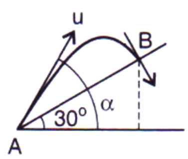
and $R = u_xT$

When a horizontal acceleration is also given to the projectile u_y and T and H will remain unchanged while the range will become

$R' = u_xT + \frac{1}{2}aT^2$

$= R + \frac{1}{2}\frac{g}{4}\left(\frac{4u_y^2}{g^2}\right) = R + H$

18. t_{AB} = time of flight of projectile $= \frac{2u\sin(\alpha - 30^o)}{g\cos 30^o}$



Now component of velocity along the plane becomes zero at point B.

$\therefore$ 0 = u cos(α – 30º) – gsin30º × T

or ucos(α – 30º)

$= g\sin 30^o \times \frac{2u\sin(\alpha - 30^o)}{g\cos 30^o}$

or $\tan(\alpha - 30^o) = \frac{\cot 30^o}{2} = \frac{\sqrt{3}}{2}$

$\therefore \quad \alpha = 30^o + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

19. Horizontal component of velocity,

$u_H = u\cos 60^o = \frac{u}{2}$

$\therefore \quad AC = u_H \times t = \frac{ut}{2}$

and AB = AC sec 30º

$$= \left(\frac{ut}{2}\right)\left(\frac{2}{\sqrt{3}}\right) = ut/\sqrt{3}$$

20. For the projectile to pass through (30 m, 40 m)

$$= 40 = 30\tan\alpha - \frac{g(30)^2}{2u^2}(1+\tan^2\alpha)$$

or $900\tan^2\alpha - 6u^2\tan\alpha + (900 + 8u^2) = 0$

For real value of α,

$(6u^2)^2 \geq 3600\,(900 + 8u^2)$

or $(u^4 - 800u^2) \geq 900{,}00$

or $(u^2 - 400)^2 \geq 25{,}000$

or $u^2 \geq 900$

or $u \geq 30$ m/s.

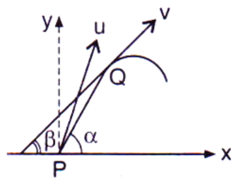
21. Let P be the position of projectile when it is moving with velocity u at an angle to the horizon and Q the position when the direction of path makes an angle β with horizontal.

Take P as origin and horizontal and vertical lines through it as axes.

Suppose time from P to Q is t, then v is the velocity at Q.

hence $v = \cos\beta = u\cos\alpha$ ....(i)

$v\sin\beta = u\sin\alpha - \sin\alpha - gt$ .....(ii)



Dividing equation (ii) by equation (i)

$$\tan\beta = \frac{u\sin\alpha - gt}{u\cos\alpha}$$

$$t = \frac{u\cos\alpha(\tan\alpha - \tan\beta)}{g}$$ ....(iii)

If in the position Q, the angle turned through be θ, then $\alpha - \beta = \theta$

or $\beta = \alpha - \theta$

From eqn. (iii), $t = \frac{u\cos\alpha}{g}[\tan ga - \tan(\alpha - \theta)]$

$$= \frac{u\cos\alpha}{g}\left[\frac{\sin\alpha}{\cos\alpha} - \frac{\sin(\alpha-\theta)}{\cos(\alpha-\theta)}\right]$$

$$= \frac{u\cos\alpha}{g}\frac{\sin\theta}{\cos\alpha\cos(\alpha-\theta)}$$

$$= \frac{u\sin\theta}{g\cos(\theta-\alpha)}$$

22. $100 = (V_t - V_m)10$, $V_t = 15$ m/s

In second case : $100 = (V_t + V_m)t$

From 1st equation : $V_m = 5$ m/s

From 2nd equation : t = 5 sec

23.

24. Time taken by body A, $t_1 = 5$ sec

Acceleration of body A = a_1

Time taken by body B, $t_2 = 5 - 2 = 3$ sec

Acceleration of body B = a_2

Distance covered by first body in 5th second after start,

$$s_5 = u + \frac{a_1}{2}(2t_1 - 1)$$

$$= 0 + \frac{a_1}{2}(2\times 5 - 1) = \frac{9a_1}{2}$$

Distance covered by the second body in the 3rd second after is start,

$$s_3 = u + \frac{a_2}{2}(2t_2 - 1)$$

$= 0 + \frac{a_2}{2}(2 \times 3 - 1) = \frac{5a_2}{2}$

Since, $s_5 = s_3$

$\therefore \quad \frac{9a_1}{2} = \frac{5a_2}{2}$ or $a_1 : a_2 = 5 : 9$

25. $\overset{P}{\underset{v_p}{\longrightarrow}} \overset{Q}{\underset{v_Q}{\longrightarrow}}$ $v_P - v_Q = 2.4$ m/s

$\overset{P}{\underset{v_P}{\longrightarrow}} \overset{Q}{\underset{v_Q}{\longleftarrow}}$ $v_P + v_Q = 6.0$ m/s

$\therefore$ $v_P = 4.2$ m/s; $v_Q = 1.8$ m/s

26. For the motion of first ball,

u = 0, a = g, t = 3s.

Let S_1 be the distance covered by the first ball in 3 sec.

$\therefore \quad S_1 = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 10 \times (3)^2 = 45 \text{ m}$

Let S_2 be the distance covered by the second ball in 2 sec. Then

$S_2 = 0 + \frac{1}{2} \times 10 \times (2)^2 = 20 \text{ m}$

$\therefore$ Separation between the two balls

$= S_1 - S_2 = 45 - 20 = 25$ m.

27.

28. **Given** : At time t = 0, velocity, v = 0.

Acceleration, $f = f_0\left(1 - \frac{t}{T}\right)$

At f = 0, $0 = f_0\left(1 - \frac{t}{T}\right)$

Since, f_0 is a constant, $\therefore 1 - \frac{t}{T} = 0$ or t = T

Also, acceleration, $f = \frac{dv}{dt}$

$\therefore \quad \int_0^{v_x} dv = \int_{t=0}^{t=T} f dt = \int_0^T f_0\left(1 - \frac{t}{T}\right)dt$

$\therefore \quad v_x = \left[f_0 t - \frac{f_0 t^2}{2T}\right]_0^T = f_0 T - \frac{f_0 T}{2T} = \frac{1}{2} f_0 T$

29. $S_2 = \frac{1}{2}gt_2^2 = \frac{10}{2} \times (3)^2 = 45 \text{ m}$

$S_1 = \frac{1}{2}gt_1^2 = \frac{10}{2} \times (5)^2 = 125 \text{ m}$

$\therefore$ S – S = 125 – 45 = 80 m

30. $S = ut + \frac{1}{2}gt^2$

$30 = -25t + \frac{10}{2}t^2$ or $t^2 - 5t - 6 = 0$

or (t – 6) (t + 1) = 0

$\therefore$ t = 6 sec

[CHEMISTRY]

43.

$$[H^+] = 10^{-pH}$$
$$C\alpha = 10^{-5}$$
$$\alpha = \frac{10^{-5}}{C} = 10^{-5}$$
$$K_a = C\alpha^2 = \mathbf{1 \times 10^{-10}}$$

44.

Lower the pK_b, stronger is the base.

45.

Lower the pK_a, stronger is the acid.

46.

$$HQ \rightleftharpoons H^+ + Q^-$$

	HQ	H⁺	Q⁻
At $t = 0$	0.1	–	–
At Eqm.	$(0.1-x)$	x	x

$$pH = 3 \Rightarrow [H^+] = x = 10^{-3}$$
$$0.1 - x \simeq 0.1$$
$$K_a = \frac{x \cdot x}{0.1 - x} = \frac{10^{-3} \times 10^{-3}}{0.1} = \mathbf{10^{-5}}$$

Alternatively :

$$x = c\,\alpha = 10^{-3}$$
$$\alpha = \frac{10^{-3}}{0.1} = 10^{-2}$$
$$K_a = c\,\alpha^2 = 0.1 \times 10^{-4} = \mathbf{10^{-5}}$$

47.

$$[OH^-] = \frac{100 \times 0.2 - 100 \times 0.1}{1000} = 10^{-2}$$
$$\Rightarrow \quad pOH = 2 \Rightarrow pH = \mathbf{12}$$

48.

Change of pH from 1 to 2 $\Rightarrow$ change in $[H^+]$ from $10^{-1}\,M$ to $10^{-2}\,M$

$$M_2V_2\,(\text{dil.}) = M_1V_1\,(\text{conc.})$$
$$V_2 = \frac{10^{-1} \times 1}{10^{-2}} = 10\,L$$

Volume of H_2O added = 10 – 1 = **9L**

49.

CH_3COOH is a weak acid. $10^{-2}M$ CH_3COOH will give much less $[H^+]$ concentration than $10^{-2}\,M$. Hence, pH will be **more than 2.**

50.

$$\frac{\text{strength of acid } A}{\text{strength of acid } B} = \sqrt{\frac{(K_a)A}{(K_a)B}}$$

pK_a of $A = 4 \Rightarrow K_a = 10^{-4}$

pK_a of $B = 5 \Rightarrow K_a = 10^{-5}$

$$\frac{\text{strength of acid } A}{\text{strength of acid } B} = \sqrt{\frac{10^{-4}}{10^{-5}}} = \sqrt{10} = \mathbf{3.2}$$

51.

$pH = 3 \Rightarrow [H^+] = 10^{-3} M$

On dilution, $[H^+] = \frac{1}{2} \times 10^{-3} = 5 \times 10^{-4} M$

New $pH = -\log(5 \times 10^{-4}) = -(0.699 - 4) = \mathbf{3.301}$

52.

Sodium butyrate C_4H_9COONa is salt of weak acid and strong base.

$$pH = -\frac{1}{2}[\log K_w + \log K_a - \log C]$$
$$= -\frac{1}{2}[\log 10^{-14} + \log(2 \times 10^{-5}) - \log(0.2)]$$
$$= -\frac{1}{2}[-14 + \log 2 - 5 - \log \frac{1}{5}]$$
$$= -\frac{1}{2}[-14 + 0.301 - 5 + 0.699] = \mathbf{9.0}$$

53.

$$MX_4 \rightleftharpoons \underset{S}{M} + \underset{4S}{4X}$$ (S is solubility in mol L^{-1})

$K_{sp} = (S)(4S)^4 = 256 S^5$

$$S = \left[\frac{K_{sp}}{256}\right]^{1/5}$$

54.

$$MX_2 \rightleftharpoons \underset{S}{M^{2+}} + \underset{2S}{2X^-}$$ (S is solubility in mol L^{-1})

$K_{sp} = S \times (2S)^2 = 4S^3$

$4S^3 = 4 \times 10^{-12}$

$S = \mathbf{1 \times 10^{-4}}$ M

55.

$K_{sp} = [Ag^+][IO_3^-] = S^2$

Solubility 'S' $= \sqrt{1 \times 10^{-8}} = 1 \times 10^{-4} M = 1 \times 10^{-4} \times 283\ gL^{-1}$

Weight of $AgIO_3$ in 100 mL solution $= 283 \times 10^{-5}$ g

$= \mathbf{2.83 \times 10^{-3}}$ **g**

56.

K_{sp} of $Cr(OH)_3 = S \times 3^3 S^3$

$27S^4 = 1.6 \times 10^{-30}$

$S = \sqrt[4]{1.6 \times 10^{-30} / 27}$

57.

$A_3B_2 \rightleftharpoons \underset{3x}{3A} + \underset{2x}{2B}$ [for solubility of A_3B_2 as xM]

$K_{sp} = [A]^3 \times [B]^2 = [3x]^3 \times [2x]^2 = \mathbf{108\,x^5}$

58.

$CaF_2 \rightleftharpoons Ca^{2+} + 2F^-$

For solubility 'S', $K_{sp} = (S)(2S)^2 = 4S^3$ (S is solubility)

$4S^3 = 3.2 \times 10^{-11}$

$S^3 = 8 \times 10^{-12}$

$S = \mathbf{2 \times 10^{-4}\,M}$

59.

For solubility S, K_{sp} of $A_2B_3 = (2)^2 \times (3)^3 \times S^2 \times S^3 = 108 \times (1 \times 10^{-2})^5$

$= 108 \times 10^{-10} = \mathbf{1.08 \times 10^{-8}}$

60.

$HgCl_2 \rightleftharpoons Hg^{2+} + 2Cl^-$

$K_{sp} = S \times (2S)^2 = 4S^3$ (S is solubility)

$4S^3 = 4 \times 10^{-15}$

$S = 10^{-5}$

$[Cl^-] = 2S = \mathbf{2 \times 10^{-5}\,M}$

[MATHEMATICS]

73. $\cos\left[\cos^{-1}\left(\frac{7}{25}\right)\right] = \frac{7}{25}$ as $\cos(\cos^{-1} x) = x\ \forall x \in [-1, 1]$

74. $\sin^{-1}\left(\sin\frac{5\pi}{3}\right) = -2\pi + \frac{5\pi}{3}$

$\because \frac{5\pi}{3} \in \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right) = \frac{\pi}{3}$. Where $\sin^{-1}\sin x = -2\pi + x$

75. $\because$ $\sin^{-1}(\sin x) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos^{-1}(\cos y) \in [0, \pi]$

$\sec^{-1}(\sec x) \in [0, \pi] \sim \{\pi/2\}$, therefore $\sin^{-1}(\sin x) + \cos^{-1}$

$(\cos y) + \sec^{-1}(\sec z) = \frac{5\pi}{2}$ possible if

$\sin x = 1$, $\cos y = -1$, and $\sec z = -1$

$\Rightarrow$ $\sin x + \cos y + \sec z = -1$

76. Let $\sin\left(\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{3}\right)=\sin(\theta-\phi);$

Where $\theta=\sin^{-1}\frac{1}{2} \Rightarrow \sin\theta=\frac{1}{2}$

$\phi=\cos^{-1}\frac{1}{3}$ and $\cos\phi=\frac{1}{3}$

$=\sin\theta\cos\phi+\cos\theta\sin\phi=\frac{1}{2}.\frac{1}{3}+\frac{\sqrt{3}}{2}\frac{2\sqrt{2}}{3}$

$\because$ $\theta\in\left(0,\frac{\pi}{2}\right)$ and $\theta\in(0,\pi)=\frac{1+2\sqrt{6}}{6}$

77. Let $\sin^{-1}x=\theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\Rightarrow$ $\cos(2\theta)=\frac{1}{9}$ $\Rightarrow$ $1-2\sin^2\theta=\frac{1}{9}$

$\Rightarrow$ $2(\sin(\sin^{-1}x))^2=\frac{8}{9} \Rightarrow$ $x^2 = 4/9$

$\Rightarrow$ $x = \pm 2/3$

78. Let $\tan^{-1}(1/5)=\theta$ $\Rightarrow$ $\cos\theta=\frac{1}{5}$,

$\tan\left[2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right]=\tan\left(2\theta-\frac{\pi}{4}\right)=\frac{\tan 2\theta-1}{1+\tan 2\theta}$

$=\frac{\frac{5}{12}-1}{1+\frac{5}{12}}=\frac{-7}{17}$ $\because \tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}=\frac{2}{5}\times\frac{25}{27}=\frac{5}{12}$

79. Let $\cos^{-1}\left(\frac{1}{5}\right)=\theta$ $\Rightarrow \cos\theta=\frac{1}{5}$,

$\sin\left(\frac{\pi}{2}-\theta\right)=\frac{1}{5}$ $\Rightarrow$ $\frac{\pi}{2}-\theta=\sin^{-1}\frac{1}{5}$

$\cos\left[2\cos^{-1}\frac{1}{5}+\sin^{-1}\frac{1}{5}\right]=\cos\left[2\theta+\frac{\pi}{2}-\theta\right]$

$=\cos\left[\frac{\pi}{2}+\theta\right]=-\sin\theta$ $\because \theta\in\left(0,\frac{\pi}{2}\right)$

$=-\sqrt{1-\cos^2\theta}=-\sqrt{1-\frac{1}{25}}=-\sqrt{\frac{24}{25}}=\frac{-2\sqrt{6}}{5}$

80. $x^2 - x - \pi + \sin^{-1}(\sin 2) < 0$

$\because$ $2\in\left(\frac{\pi}{2},\pi\right); \sin^{-1}(\sin 3)=\pi-2$

Thus, the above inequality reduces to $x^2 - x - 2 < 0$

$\Rightarrow$ $(x-2)(x+1) < 0 \Rightarrow x \in (-1, 2)$

81. Let $\sin^{-1} x = \theta \in \left[-\frac{\pi}{2}, 0\right)$ as $x \in [-1, 0)$

$\Rightarrow$ $x = \sin\theta$

$\Rightarrow$ $\cos^{-1}(2x^2 - 1) - 2\sin^{-1} x = \cos^{-1}(\cos 2\theta) - 2\theta$

$= \pi - \cos^{-1}(\cos 2\theta) - 2\pi = \pi - (2\theta) - 2\theta = \pi$

82. $\because$ $x > 1$ thus $\theta = \tan^{-1} x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\Rightarrow$ $2\tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\theta + \sin^{-1}(\sin 2\theta)$

$2\theta \in \left(\frac{\pi}{2}, \pi\right) = 2\theta + \pi - 2\theta - \pi$

83. $\because$ $\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$

$\Rightarrow$ $\cos^{-1} x = \frac{\pi}{2} - \frac{\pi}{5} = \frac{5\pi - 2\pi}{10} = \frac{3\pi}{10}$

84. $\sin^{-1}\left(\frac{4}{5}\right) + 2\tan^{-1}\left(\frac{1}{3}\right) = \sin^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{1-\frac{1}{9}}{1+\frac{1}{9}}\right)$;

as $\frac{1}{3} > 0 = \sin^{-1}\frac{4}{5} + \cos^{-1}\left(\frac{8}{10}\right) = \sin^{-1}\frac{4}{5} + \cos^{-1}\frac{4}{5} = \frac{\pi}{2}$

85. Given $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

Applying tan on both side $\tan(\tan^{-1} 2x + \tan^{-1} 3x) = 1$

$\Rightarrow$ $\frac{2x + 3x}{1 - (2x)(3x)} = 1$ $\Rightarrow$ $5x = 1 - 6x^2$

$\Rightarrow$ $6x^2 + 5x - 1 = 0$

$\Rightarrow$ $6x^2 + 6x - x - 1 = 0 \Rightarrow 6x(x + 1) - (x + 1) = 0$

$\Rightarrow$ $(6x - 1)(x + 1) = 0$

$\Rightarrow$ $x = 1/6, -1$ but $x = -1$ does not satisfy the equation as L.H.S. becomes negative

$\Rightarrow$ $x = 1/6$

86. $\tan^{-1} x + \tan^{-1}\frac{1}{y} = \tan^{-1} 3$

$\Rightarrow$ $\frac{xy + 1}{y - x} = 3$ or, $y = \frac{3x + 1}{3 - x}$

$y > 0 \Rightarrow \frac{3x + 1}{3 - x} > 0 \Rightarrow -\frac{1}{3} < x < 3$

$\therefore$ x = 1, 2 and corresponding values of y are 2, y. Hence two pairs (1, 2) and (2, 7) are possible

87. $k = \cos^{-1} x + \cos^{-1}\left[\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right] = \frac{\pi}{3}$

$\therefore$ $-1 \le x \le 1$, also $3 - 3x^2 \ge 0$

$-1 \le \frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2} \le 1$

$\Rightarrow$ $-1 \le x \le 1$

$0 \le \frac{x}{2} + 1 + \frac{1}{2}\sqrt{3-3x^2} \le 1$ ....(2)

Solving (2) we get option (c)

88. $\cos^{-1}(\cos 12) - \sin^{-1}(\sin 14) \Rightarrow 12 - 14 = -2$

89. $\angle A = 90^o$

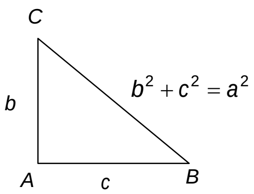
$\tan^{-1}\left(\frac{c}{a+b}\right) + \tan^{-1}\left(\frac{b}{a+c}\right)$

$= \tan^{-1}\left[\frac{\frac{c}{a+b} + \frac{b}{a+c}}{1 - \left(\frac{c}{a+b}\right)\left(\frac{b}{a+c}\right)}\right]$

$= \tan^{-1}\left[\frac{ca + c^2 + ab + b^2}{a^2 + ab + ca + bc - bc}\right]$

$= \tan^{-1}\left[\frac{a^2 + ab + ca}{a^2 + ab + ca}\right]$

$= \tan^{-1}(1) = \frac{\pi}{4}$



90 $\cos^{-1} x + \cos^{-1}(2x) = -\pi \Rightarrow \cos^{-1} 2x = -\pi - \cos^{-1} x$

$\Rightarrow 2x = \cos(\pi + \cos^{-1} x)$

$2x = \cos\pi(\cos\cos^{-1} x) - \sin\pi\sin(\cos^{-1} x)$

$2x = -x \Rightarrow x = 0$

But $x = 0$ does not satisfy the given equation.

No solution will exist